

Roll No LHR-G1-12-18 (To be filled in by the candidate)

(Academic Sessions 2014 – 2016 to 2016 – 2018)

MATHEMATICS

218-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – II (Objective Type)

GROUP – I

Maximum Marks : 20

PAPER CODE = 8197

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	If $y = \sqrt{1-x^2}$, $0 < x < 1$ then $\frac{dy}{dx} = :$ (A) $\sqrt{x^2-1}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{x}{\sqrt{1-x^2}}$ (D) $\frac{-x}{\sqrt{1-x^2}}$
2	$\int 3^x dx = :$ (A) $3^x + c$ (B) $3^x \ln 3 + c$ (C) $\frac{3^x}{\ln 3} + c$ (D) $3 \ln 3^x + c$
3	$\int_0^{\frac{\pi}{2}} \cos x dx = :$ (A) 0 (B) 1 (C) 2 (D) 3
4	If $f(x)$ has second derivative at "c" such that $f'(c) = 0$ and $f''(c) < 0$ then "c" is a point of : (A) Maxima (B) Minima (C) Zero point (D) Point of inflection
5	If $y = e^{\sin x}$, then $\frac{dy}{dx} = :$ (A) $e^{\sin x}$ (B) $e^{\sin x} \cos x$ (C) $e^{\sin x} + \cos x$ (D) $-e^{\sin x} \cos x$
6	$\cosh^2 x - \sinh^2 x = :$ (A) 1 (B) -1 (C) 0 (D) 2
7	$\frac{d}{dx} \sin^{-1} x = :$ (A) $\frac{1}{\sqrt{1+x^2}}$ (B) $\cos^{-1} x$ (C) $\frac{1}{\sqrt{1-x^2}}$ (D) $\frac{1}{\sqrt{1-x}}$
8	$\int \frac{1}{f(x)} \times f'(x) dx = :$ (A) $\ln x + c$ (B) $\ln[f'(x) + c]$ (C) $\frac{1}{f(x)} + c$ (D) $\ln f(x) + c$
9	The order of the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2x = 0$ is : (A) 2 (B) 1 (C) 0 (D) 3

(Turn Over)

LHR-91-12-19 (2)

1-10	Let $f(x) = x^2 + \cos x$, then $f(x)$ is : (A) Odd function (B) Constant function (C) Even function (D) Neither even nor odd
11	The centroid of a triangle divides each median in ratio : (A) 2 : 1 (B) 1 : 2 (C) 2 : 3 (D) 1 : 1
12	The straight line $y = mx + c$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if : (A) $c^2 = a^2 m^2 - b^2$ (B) $c^2 = b^2 m^2 + a^2$ (C) $c^2 = b^2 m^2 - a^2$ (D) $c^2 = a^2 m^2 + b^2$
13	The perpendicular distance of line $3x + 4y - 10 = 0$ from the origin is : (A) 0 (B) 1 (C) $\frac{1}{2}$ (D) 2
14	Axis of the parabola $x^2 = 4ay$ is : (A) $y = 0$ (B) $x = 0$ (C) $x = y$ (D) $x = 1$
15	If α is the inclination of the line ℓ then $\frac{x - x_1}{\cos \alpha} = \frac{y - y_1}{\sin \alpha} = r$ (say) is called : (A) Point slope form (B) Normal form (C) Symmetric form (D) Intercept form
16	The direction cosines of y-axis are : (A) (0, 1, 0) (B) (1, 0, 0) (C) (0, 0, 1) (D) (0, 0, 0)
17	If α is the inclination of a line " ℓ " then it must be true that : (A) $0 \leq \alpha < \frac{\pi}{2}$ (B) $\frac{\pi}{2} \leq \alpha < \pi$ (C) $0 \leq \alpha < \pi$ (D) $0 \leq \alpha < 2\pi$
18	The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle with centre : (A) $(-g, -f)$ (B) $(-f, +g)$ (C) (f, g) (D) $(0, 0)$
19	Length of the vector $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ is : (A) 2 (B) 4 (C) 3 (D) 5
20	The feasible solution which maximizes or minimizes the objective function is called : (A) Exact solution (B) Optimal solution (C) Final solution (D) Objective solution

SECTION – I

2. Write short answers to any EIGHT (8) questions :

LHR-G1-12-18

16

- (i) State sandwich theorem.
- (ii) Express the area " A " of a circle as a function of its circumference " C ".
- (iii) If $f(x) = \begin{cases} x+2, & x \leq -1 \\ c+2, & x > -1 \end{cases}$, find " c " so that $\lim_{x \rightarrow -1} f(x)$ exists
- (iv) Define differentiation.
- (v) Differentiate $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$ w.r.t x
- (vi) Find $\frac{dy}{dx}$ if $xy + y^2 = 0$
- (vii) Find $\frac{dy}{dx}$ if $y = x \cos y$
- (viii) Prove that $\frac{d}{dx} (\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$, $x \in (-1, 1)$
- (ix) Find $\frac{dy}{dx}$ if $y = x e^{\sin x}$
- (x) Define power series.
- (xi) Find extreme values for $f(x) = x^2 - x - 2$
- (xii) Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{x}{2}\right)$

3. Write short answers to any EIGHT (8) questions :

16

- (i) Find $\frac{dy}{dx}$ using differentials if $xy - \log_e x = c$
- (ii) Evaluate the integral $\int \frac{x}{x+2} \cdot dx$
- (iii) Evaluate the integral $\int \frac{1}{a^2 - x^2} \cdot dx$
- (iv) Evaluate the integral $\int x \sin x \cos x dx$
- (v) Evaluate the integral $\int x^2 e^{ax} \cdot dx$
- (vi) Evaluate the integral $\int e^{3x} \left(\frac{3 \sin x - \cos x}{\sin^2 x} \right) dx$
- (vii) Prove that $\int_a^b f(x) \cdot dx = - \int_b^a f(x) \cdot dx$
- (viii) Evaluate the definite integral $\int_0^3 \frac{dx}{x^2 + 9}$
- (ix) Find the area bounded by cos function from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$

3. (x) Solve the differential equation $\sin y \operatorname{cosec} x \frac{dy}{dx} = 1$
 (xi) Define optimal solution and feasible solution.
 (xii) Graph the region indicated by $4x - 3y \leq 12$, $x \geq -\frac{3}{2}$

18

4. Write short answers to any NINE (9) questions :

- (i) Show that the points A (3, 1), B (-2, -3) and C (2, 2) are vertices of an isosceles triangle.
 (ii) Find an equation of a line through the points (-2, 1) and (6, -4)
 (iii) Find an equation of the line bisecting the first and third quadrants.
 (iv) Find an equation of the line with x-intercept : -3 and y - intercept : 4
 (v) Convert $2x - 4y + 11 = 0$ into slope intercept form.
 (vi) Write an equation of the parabola with focus (-1, 0), vertex (-1, 2)
 (vii) Find the focus and directrix of the parabola $y = 6x^2 - 1$
 (viii) Find an equation of the ellipse with centre (0, 0), focus (0, -3), vertex (0, 4)
 (ix) Find the eccentricity and directrices of the ellipse whose equation is $25x^2 + 9y^2 = 225$
 (x) Define unit vector.
 (xi) Find a unit vector in the direction of the vector $\underline{v} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$
 (xii) Find a vector whose magnitude is '4' and is parallel to $2\underline{i} - 3\underline{j} + 6\underline{k}$
 (xiii) Find a scalar "α" so that the vectors $2\underline{i} + \alpha\underline{j} + 5\underline{k}$ and $3\underline{i} + \underline{j} + \alpha\underline{k}$ are perpendicular.

SECTION - II

Note : Attempt any THREE questions.

5. (a) If $f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$ 5
 Find value of k so that f is continuous at $x = 2$
 (b) Show that $y = x^x$ has maximum value at $x = \frac{1}{e}$ 5
 6. (a) Evaluate $\int e^{2x} \cos 3x dx$ 5
 (b) The three points A (7, -1), B (-2, 2) and C (1, 4) are consecutive vertices of a parallelogram, find the fourth vertex. 5
 7. (a) Find the area bounded by the curve $y = x^3 - 4x$ and x-axis. 5
 (b) Minimize $z = 2x + y$ subject to the constraints $x + y \geq 3$, $7x + 5y \leq 35$, $x \geq 0$, $y \geq 0$ 5
 8. (a) Find the condition that the line $y = mx + c$ touches the circle $x^2 + y^2 = a^2$ at a single point. 5
 (b) Find x so that points A (1, -1, 0), B (-2, 2, 1) and C (0, 2, x) form triangle with right angle at C. 5
 9. (a) Find the centre, foci, eccentricity, vertices and equations of directrices of $\frac{y^2}{4} - x^2 = 1$ 5
 (b) Find volume of the tetrahedron with the vertices A (2, 1, 8), B (3, 2, 9), C (2, 1, 4) and D (3, 3, 10) 5

No LHR-G2-12-18 (To be filled in by the candidate)

(Academic Sessions 2014 – 2016 – 2018)

MATHEMATICS

218-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – II (Objective Type)

GROUP – II

Maximum Marks : 20

PAPER CODE = 8198

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	$\frac{d}{dx} \log_a x = :$ (A) $\frac{1}{x}$ (B) $x \ln x - x$ (C) $\frac{1}{x} \ln a$ (D) $\frac{1}{x \ln a}$
2	$\int \sin x \cos x dx :$ (A) $\frac{1}{2} \cos 2x$ (B) $-\frac{1}{2} \cos 2x$ (C) $\frac{\sin^2 x}{2}$ (D) $\frac{\cos^2 x}{2}$
3	$\int \frac{1}{x\sqrt{x^2-1}} dx :$ (A) $\sin^{-1} x$ (B) $\tan^{-1} x$ (C) $\sec^{-1} x$ (D) $\operatorname{cosec}^{-1} x$
4	If $x = f(\theta), y = g(\theta)$ then $\frac{dy}{dx} :$ (A) $\frac{dy}{d\theta} \frac{d\theta}{dx}$ (B) $\frac{dx}{d\theta} \frac{d\theta}{dy}$ (C) $\frac{d\theta}{dy} \frac{dx}{d\theta}$ (D) $\frac{dy}{d\theta} \frac{dx}{d\theta}$
5	$\frac{d}{dx} \sec hx = :$ (A) $\sec hx \tanh x$ (B) $-\sec hx \tanh x$ (C) $\tan h^2 x$ (D) $\sec h^2 x$
6	If at least one vertical line meets the curve at more than two points then curve is : (A) A function (B) Not a function (C) One – to – one function (D) Onto function
7	$\frac{d}{dx} \cosh x = :$ (A) $-\sin hx$ (B) $\sec hx$ (C) $-\sec hx$ (D) $\sin hx$
8	$\int \sec^2 x dx :$ (A) $\tan x$ (B) $\frac{\sec^3 x}{3}$ (C) $\tan^2 x$ (D) $\sec x \tan x$
9	Solution of $\frac{dy}{dx} = \frac{-y}{x}$ is : (A) $\frac{x}{y} = c$ (B) $\frac{y}{x} = c$ (C) $y = cx$ (D) $xy = c$

(Turn Over)

1-10	Domain of $f(x) = x^2 + 1$: (A) \mathbb{R} (B) $\mathbb{R} - \{1\}$ (C) $\mathbb{R} - \{-1\}$ (D) $[1, \infty)$
11	Equation of line bisecting II and IV quadrant : (A) $y = x$ (B) $y = -x$ (C) $y = \frac{1}{x}$ (D) $x + y = 1$
12	Set of all points equidistant from a fixed point form : (A) Ellipse (B) Parabola (C) Hyperbola (D) Circle
13	Joint equation of two lines is $ax^2 + 2hxy + by^2 = 0$, if θ is angle between them, then $\tan \theta =$: (A) $\frac{2\sqrt{h^2 + ab}}{a + b}$ (B) $\frac{2\sqrt{h^2 - ab}}{a + b}$ (C) $\frac{\sqrt{h^2 + ab}}{a + b}$ (D) $\frac{\sqrt{h^2 - ab}}{a + b}$
14	Focal chord perpendicular to axis of parabola is called : (A) Latus Rectum (B) Eccentricity (C) Vertex (D) Axis
15	Horizontal line through $(7, -9)$ is : (A) $x = 7$ (B) $x = -9$ (C) $y = 7$ (D) $y = -9$
16	Projection of vector \vec{u} on vector \vec{v} is : (A) $\frac{\vec{u} \cdot \vec{v}}{ \vec{v} }$ (B) $\frac{\vec{u} \cdot \vec{v}}{ \vec{u} }$ (C) $\frac{\vec{u} \times \vec{v}}{ \vec{v} }$ (D) $\frac{\vec{u} \times \vec{v}}{ \vec{u} }$
17	Distance of (x_1, y_1) from line $ax + by + c = 0$ is : (A) $\frac{ ax_1 + by_1 + c }{\sqrt{a^2 + b^2}}$ (B) $\frac{ ax_1 + by_1 - c }{\sqrt{a^2 + b^2}}$ (C) $\frac{ ax_1 + by_1 + c }{\sqrt{a + b}}$ (D) $\frac{ ax_1 + by_1 - c }{\sqrt{a + b}}$
18	For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, ($a > b$) then eccentricity $e =$: (A) $\frac{\sqrt{a^2 - b^2}}{a}$ (B) $\frac{\sqrt{a^2 + b^2}}{a}$ (C) $\frac{\sqrt{b^2 - a^2}}{a}$ (D) $\frac{\sqrt{b^2 - a^2}}{b}$
19	If \vec{v} is any vector then vector of magnitude 5 opposite to \vec{v} is : (A) $5\vec{v}$ (B) $-5\vec{v}$ (C) $5\frac{\vec{v}}{ \vec{v} }$ (D) $-5\frac{\vec{v}}{ \vec{v} }$
20	System of linear inequalities involved in the problem is called : (A) Coefficients (B) Solution (C) Problem constraints (D) Boundaries

SECTION – I

2. Write short answers to any EIGHT (8) questions :

LHR-G2-12-18

16

- (i) Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x$
- (ii) Determine whether function $f(x) = \frac{x^3 - x}{x^2 + 1}$ is even or odd.
- (iii) Evaluate $\lim_{x \rightarrow 0} \frac{\sec x - \cos x}{x}$
- (iv) Find $\frac{dy}{dx}$ if $y = \frac{a+x}{a-x}$
- (v) Find $\frac{dy}{dx}$ if $x^2 - 4xy - 5y = 0$
- (vi) Differentiate $x^2 - \frac{1}{x^2}$ w.r.t x^4
- (vii) Differentiate $\sin^{-1} \sqrt{1-x^2}$ w.r.t x
- (viii) Find $\frac{dy}{dx}$ if $y = \ln(x + \sqrt{x^2 + 1})$
- (ix) Find $\frac{dy}{dx}$ if $y = e^{-2x} \sin 2x$
- (x) Find $\frac{d^2y}{dx^2}$ if $y^3 + 3ax^2 + x^3 = 0$
- (xi) Find y_2 if $y = \cos^3 x$
- (xii) Find $\frac{dy}{dx}$ if $y = \ln \left(\frac{x^2 - 1}{x^2 + 1} \right)^{\frac{1}{2}}$

3. Write short answers to any EIGHT (8) questions :

16

- (i) Find δy and dy : $y = \sqrt{x}$, when x changes from 4 to 4.41
- (ii) Evaluate $\int \frac{e^{2x} + e^x}{e^x} dx$
- (iii) Evaluate $\int (a - 2x)^{\frac{3}{2}} dx$
- (iv) Evaluate $\int \frac{x+b}{(x^2 + 2bx + c)^{\frac{1}{2}}} dx$
- (v) Evaluate $\int xe^x dx$
- (vi) Evaluate $\int e^x \left(\frac{1}{x} + \ln x \right) dx$
- (vii) Evaluate $\int_{-1}^3 (x^3 + 3x^2) dx$
- (viii) Evaluate $\int_0^{\frac{\pi}{3}} \cos^2 \theta \sin \theta d\theta$

3. (ix) Find the area between the x-axis and the curve $y = 4x - x^2$ from $x = 0$ to $x = 4$
 (x) Define differential equation.
 (xi) Solve $\frac{dy}{dx} = \frac{y^2 + 1}{e^{-x}}$
 (xii) Solve $\frac{dy}{dx} = 2x$

4. Write short answers to any NINE (9) questions :

18

- (i) Write down equation of straight line with x-intercept $(2, 0)$ and y-intercept $(0, -4)$
 (ii) Find an equation of a line bisecting 2nd and 4th quadrants.
 (iii) Find an equation of a line with x-intercept : -9 and slope : -4 .
 (iv) Prove that if the lines are perpendicular, then product of their slopes $= -1$
 (v) Find the measure of angle between the lines represented by $x^2 - xy - 6y^2 = 0$
 (vi) Find focus and vertex of the parabola $y = 6x^2 - 1$
 (vii) Find equation of latus rectum of parabola $y^2 = -8(x - 3)$
 (viii) Find an equation of an ellipse with foci $(\pm 3, 0)$ and minor axis of length 10.
 (ix) Find the foci and length of the latus rectum of the ellipse $9x^2 + y^2 = 18$
 (x) Define direction angles and direction cosines of a vector.
 (xi) Find the projection of vector \underline{a} along vector \underline{b} and projection of vector \underline{b} along \underline{a}
 when $\underline{a} = \hat{i} - \hat{k}$, $\underline{b} = \hat{j} + \hat{k}$
 (xii) Find a vector perpendicular to each of the vectors $\underline{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\underline{b} = 4\hat{i} + 2\hat{j} - \hat{k}$
 (xiii) Convert $2x - 4y + 11 = 0$ into slope intercept form.

SECTION - II

Note : Attempt any THREE questions.

5. (a) Prove that $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$ 5
 (b) Prove that $y \frac{dy}{dx} + x = 0$ if $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$ 5
6. (a) Show that $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln(x + \sqrt{x^2 - a^2}) + c$ 5
 (b) The points A $(-1, 2)$, B $(6, 3)$ and C $(2, -4)$ are vertices of a triangle, then show that the line joining the mid-point "D" of \overline{AB} and mid-point "E" of \overline{AC} is parallel to \overline{BC} and $\overline{DE} = \frac{1}{2} \overline{BC}$. 5
7. (a) Evaluate $\int_0^{\frac{\pi}{4}} \cos^4 t dt$ 5
 (b) Graph the feasible region of system of linear inequalities and find the corner points
 $2x + 3y \leq 18, x + 4y \leq 12, 3x + y \leq 12, x \geq 0, y \geq 0$ 5
8. (a) Find an equation of parabola having its focus at the origin and directrix parallel to y-axis. 5
 (b) Prove that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and half as long. 5
9. (a) Find the centre, foci, eccentricity, vertices and equations of directrices of $\frac{y^2}{4} - x^2 = 1$ 5
 (b) Find the value of α , in the coplanar vectors $\alpha \hat{i} + \hat{j}$, $\hat{i} + \hat{j} + 3\hat{k}$, $2\hat{i} + \hat{j} - 2\hat{k}$ 5